



Chapter 7

Stacks

CS102 Sections 51 and 52

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Spring 2008

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7-1

The Abstract Data Type: Developing an ADT During the Design of a Solution

- Specifications of an abstract data type for a particular problem
 - Can emerge during the design of the problem's solution
 - Examples
 - readAndCorrect algorithm
 - displayBackward algorithm

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Developing an ADT During the Design of a Solution

- ADT stack operations
 - Create an empty stack
 - Determine whether a stack is empty
 - Add a new item to the stack
 - Remove from the stack the item that was added most recently
 - Remove all the items from the stack
 - Retrieve from the stack the item that was added most recently

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Developing an ADT During the Design of a Solution

- A stack
 - Last-in, first-out (LIFO) property
 - The last item placed on the stack will be the first item removed
 - Analogy
 - A stack of dishes in a cafeteria

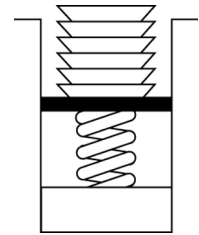


Figure 7-1
Stack of cafeteria dishes

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Developing an ADT During the Design of a Solution

- A queue
 - First in, first out (FIFO) property
 - The first item added is the first item to be removed

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Refining the Definition of the ADT Stack

- Pseudocode for the ADT stack operations


```
createStack()
// Creates an empty stack.

isEmpty()
// Determines whether a stack is empty.

push(newItem) throws StackException
// Adds newItem to the top of the stack.
// Throws StackException if the insertion is
// not successful.
```

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Refining the Definition of the ADT Stack

- Pseudocode for the ADT stack operations (Continued)

```
pop() throws StackException
// Retrieves and then removes the top of the stack.
// Throws StackException if the deletion is not
// successful.

popAll()
// Removes all items from the stack.

peek() throws StackException
// Retrieves the top of the stack. Throws
// StackException if the retrieval is not
// successful
```

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Using the ADT Stack in a Solution

- `displayBackward` and `readAndCorrect` algorithms can be refined by using stack operations
- A program can use a stack independently of the stack's implementation

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Axioms (Optional)

- Axioms are used to define an ADT formally
 - Example
 - Axiom to specify that the last item inserted into stack is the first item to be removed


```
(stack.push(newItem)).pop() = stack
```

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Simple Applications of the ADT Stack: Checking for Balanced Braces

- A stack can be used to verify whether a program contains balanced braces
 - An example of balanced braces


```
abc{defg{ijk}{l{mn}}op}qr
```
 - An example of unbalanced braces


```
abc{def}}{ghij{kl}m
```

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Checking for Balanced Braces

- Requirements for balanced braces
 - Each time you encounter a “}”, it matches an already encountered “{”
 - When you reach the end of the string, you have matched each “{”

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Checking for Balanced Braces

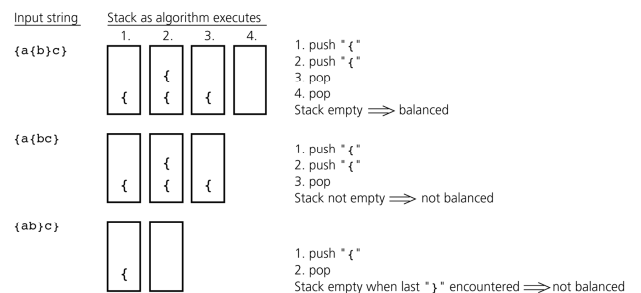


Figure 7-3

Traces of the algorithm that checks for balanced braces

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Checking for Balanced Braces

- The exception `StackException`
 - A Java method that implements the balanced-braces algorithm should do one of the following
 - Take precautions to avoid an exception
 - Provide `try` and `catch` blocks to handle a possible exception

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Recognizing Strings in a Language

- Language L
 - $L = \{w\$w' : w \text{ is a possible empty string of characters other than } \$, w' = \text{reverse}(w)\}$
 - A stack can be used to determine whether a given string is in L
 - Traverse the first half of the string, pushing each character onto a stack
 - Once you reach the \$, for each character in the second half of the string, pop a character off the stack
 - Match the popped character with the current character in the string

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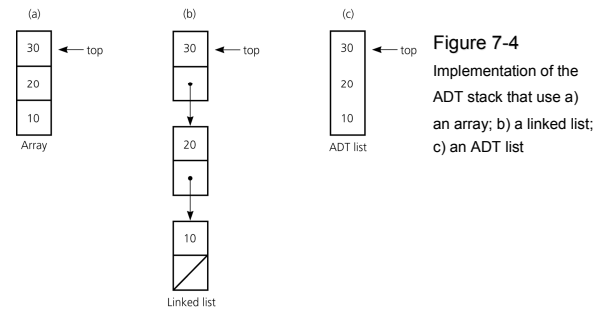
Implementations of the ADT Stack

- The ADT stack can be implemented using
 - An array
 - A linked list
 - The ADT list
- `StackInterface`
 - Provides a common specification for the three implementations
- `StackException`
 - Used by `StackInterface`
 - Extends `java.lang.RuntimeException`

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Implementations of the ADT Stack



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An Array-Based Implementation of the ADT Stack

- `StackArrayBased` class
 - Implements `StackInterface`
 - Instances
 - Stacks
 - Private data fields
 - An array of Objects called `items`
 - The index `top`

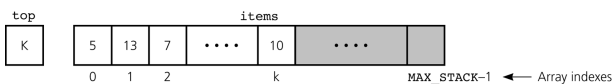


Figure 7-5
An array-based implementation

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A Reference-Based Implementation of the ADT Stack

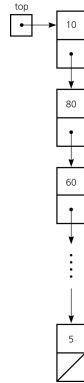
- A reference-based implementation
 - Required when the stack needs to grow and shrink dynamically
- `StackReferenceBased`
 - Implements `StackInterface`
 - `top` is a reference to the head of a linked list of items

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A Reference-Based Implementation of the ADT Stack

Figure 7-6
A reference-based implementation

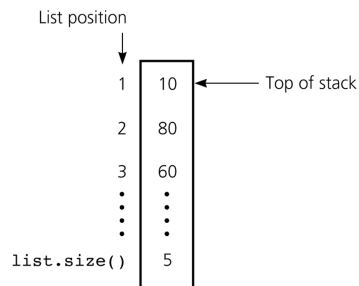


An Implementation That Uses the ADT List

- The ADT list can be used to represent the items in a stack
- If the item in position 1 of a list represents the top of the stack
 - `push(newItem)` operation is implemented as `add(1, newItem)`
 - `pop()` operation is implemented as `get(1)` followed by `remove(1)`
 - `peek()` operation is implemented as `get(1)`

An Implementation That Uses the ADT List

Figure 7-7
An implementation that uses the ADT list



Comparing Implementations

- All of the three implementations are ultimately array based or reference based
- Fixed size versus dynamic size
 - An array-based implementation
 - Uses fixed-sized arrays
 - Prevents the `push` operation from adding an item to the stack if the stack's size limit has been reached
 - A reference-based implementation
 - Does not put a limit on the size of the stack

Comparing Implementations

- An implementation that uses a linked list versus one that uses a reference-based implementation of the ADT list
 - Linked list approach
 - More efficient
 - ADT list approach
 - Reuses an already implemented class
 - Much simpler to write
 - Saves time

The Java Collections Framework Class Stack

- JCF contains an implementation of a stack class called `Stack` (generic)
- Derived from `Vector`
- Includes methods: `peek`, `pop`, `push`, and `search`
- `search` returns the 1-based position of an object on the stack

Application: Algebraic Expressions

- When the ADT stack is used to solve a problem, the use of the ADT's operations should not depend on its implementation
- To evaluate an infix expressions
 - Convert the infix expression to postfix form
 - Evaluate the postfix expression

Evaluating Postfix Expressions

- A postfix calculator
 - Requires you to enter postfix expressions
 - Example: 2, 3, 4, +, *
 - When an operand is entered, the calculator
 - Pushes it onto a stack
 - When an operator is entered, the calculator
 - Applies it to the top two operands of the stack
 - Pops the operands from the stack
 - Pushes the result of the operation on the stack

Evaluating Postfix Expressions

Key entered	Calculator action	Stack (bottom to top)
2	push 2	2
3	push 3	2 3
4	push 4	2 3 4
+	operand2 = pop stack (4) operand1 = pop stack (3)	2 3 2
	result = operand1 + operand2 (7) push result	2 2 7
*	operand2 = pop stack (7) operand1 = pop stack (2)	7 2
	result = operand1 * operand2 (14) push result	14

Figure 7-8

The action of a postfix calculator when evaluating the expression $2 * (3 + 4)$

Evaluating Postfix Expressions

- To evaluate a postfix expression which is entered as a string of characters
 - Simplifying assumptions
 - The string is a syntactically correct postfix expression
 - No unary operators are present
 - No exponentiation operators are present
 - Operands are single lowercase letters that represent integer values

Converting Infix Expressions to Equivalent Postfix Expressions

- An infix expression can be evaluated by first being converted into an equivalent postfix expression
- Facts about converting from infix to postfix
 - Operands always stay in the same order with respect to one another
 - An operator will move only “to the right” with respect to the operands
 - All parentheses are removed

Converting Infix Expressions to Equivalent Postfix Expressions

ch	stack (bottom to top)	postfixExp	
a		a	
-	-	a	
(-(a	
b	-(ab	
+	-(+	ab	
c	-(+	abc	
*	-(+*	abc	
d	-(+*	abcd	
)	-(+*	abcd*	Move operators
	-(abcd*+	from stack to
	-	abcd*+	postfixExp until " ("
/	-/	abcd*+	
e	-/	abcd*+e	Copy operators from
		abcd*+e/-	stack to postfixExp

Figure 7-9

A trace of the algorithm that converts the infix expression $a - (b + c * d)/e$ to postfix form

Application: A Search Problem

- High Planes Airline Company (HPAir)
 - Problem
 - For each customer request, indicate whether a sequence of HPAir flights exists from the origin city to the destination city

Representing the Flight Data

- The flight map for HPAir is a graph
 - Adjacent vertices
 - Two vertices that are joined by an edge
 - Directed path
 - A sequence of directed edges

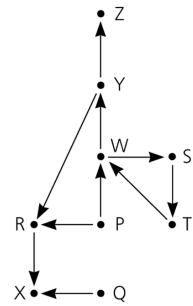


Figure 7-10
Flight map for HPAir

A Nonrecursive Solution that Uses a Stack

- The solution performs an exhaustive search
 - Beginning at the origin city, the solution will try every possible sequence of flights until either
 - It finds a sequence that gets to the destination city
 - It determines that no such sequence exists
- The ADT stack is useful in organizing an exhaustive search
- Backtracking can be used to recover from a wrong choice of a city

A Nonrecursive Solution that Uses a Stack

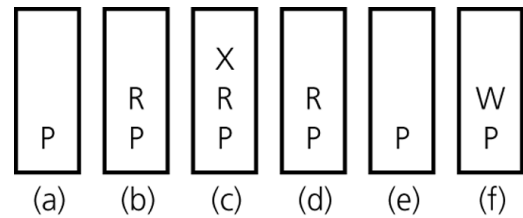


Figure 7-11
The stack of cities as you travel a) from P; b) to R; c) to X; d) back to R; e) back to P; f) to W

A Nonrecursive Solution that Uses a Stack

Action	Reason	Contents of stack (bottom to top)
Push P	Initialize	P
Push R	Next unvisited adjacent city	P R
Push X	Next unvisited adjacent city	P R X
Pop X	No unvisited adjacent city	P R
Pop R	No unvisited adjacent city	P
Push W	Next unvisited adjacent city	P W
Push S	Next unvisited adjacent city	P W S
Push T	Next unvisited adjacent city	P W S T
Pop T	No unvisited adjacent city	P W S
Pop S	No unvisited adjacent city	P W
Push Y	Next unvisited adjacent city	P W Y
Push Z	Next unvisited adjacent city	P W Y Z

Figure 7-13
A trace of the search algorithm, given the flight map in Figure 6-9

A Recursive Solution

- Possible outcomes of the recursive search strategy
 - You eventually reach the destination city and can conclude that it is possible to fly from the origin to the destination
 - You reach a city C from which there are no departing flights
 - You go around in circles

A Recursive Solution

- A refined recursive search strategy

```
searchR(originCity, destinationCity)
  Mark originCity as visited
  if (originCity is destinationCity) {
    Terminate -- the destination is reached
  }
  else {
    for (each unvisited city C adjacent to originCity) {
      searchR(C, destinationCity)
    }
  }
}
```

The Relationship Between Stacks and Recursion

- The ADT stack has a hidden presence in the concept of recursion
- Typically, stacks are used by compilers to implement recursive methods
 - During execution, each recursive call generates an activation record that is pushed onto a stack
- Stacks can be used to implement a nonrecursive version of a recursive algorithm

Summary

- ADT stack operations have a last-in, first-out (LIFO) behavior
- Algorithms that operate on algebraic expressions are an important application of stacks
- A stack can be used to determine whether a sequence of flights exists between two cities
- A strong relationship exists between recursion and stacks